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SHORT NOTE

Strain analysis using length-weighting of deformed random line elements

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Abstract—A new method of analysing strain from initially random lines is presented, which simultaneously considers their deformed length and orientation. Each line is modelled as a vector whose length is that of the line element and a relationship between the dispersion of these vectors and strain established. The method provides a simple and rapid estimate of the strain ratio for a wide variety of objects directly from their digitized shapes. The method may also remove some sampling bias in the analysis of discrete straight line elements, but its main advantage is that it can be used to analyse curved lines and outlines of objects.

INTRODUCTION

ANALYSIS of finite strain using the angular distribution of line elements was first considered by Sorby (1855) and analysed by Sanderson (1977), Harvey & Laxton (1980), and others. Most methods consider the effect of strain on lines with an initial random orientation; as strain increases these will develop an increased preferred orientation about the maximum stretch (X-axis). The methods have been extended to three-dimensions (Harvey & Laxton 1980, Sanderson & Meneilly 1981) and to initial non-uniform distributions (Sanderson 1973). These methods usually consider the change in orientation of the line independently from its change in length. The purpose of this short note is to present a simple length-weighting procedure in two-dimensions which can be used to extend the range of application of this form of strain analysis to curved line elements.

Figure 1(a) shows the essential features of the 'random line' method; initial line elements (l) are deformed to L. If we treat each line (L) as a unit vector and double its angle to put it in the range 0–360°, we can describe the preferred orientation by the normalized length of the resultant vector (R/n), where R is the length of the resultant vector and n the sample size. Sanderson (1977) examined the change in R/n with increasing strain ratio (E) and the relationship can be expressed algebraically (De Paor pers. comm.) by:

$$E = (1 + R/n)/(1 - R/n). \tag{1}$$

This method ignores the change in length of the line and, hence, requires that lines be sampled independently of their length in the deformed state. Since strain changes the length of lines it is essential that individual line elements retain their identity from the undeformed to the deformed state, and for there to be no sampling bias on the basis of length. At high strains, shortened lines at a high angle to X, may become less distinct and thus undersampled. Lengthened lines, at low angles to X, tend to become sub-parallel to each other; thus two lines may be sampled as one.

Figure 1(b) illustrates a more fundamental restriction of existing 'random line' methods, in that they cannot be applied to irregular curving lines. Small linear elements of the original curve change length with deformation. Thus random sampling of the deformed curve will produce under-representation of the shortened parts of the initial curve at a high angle to X, whereas lengthened

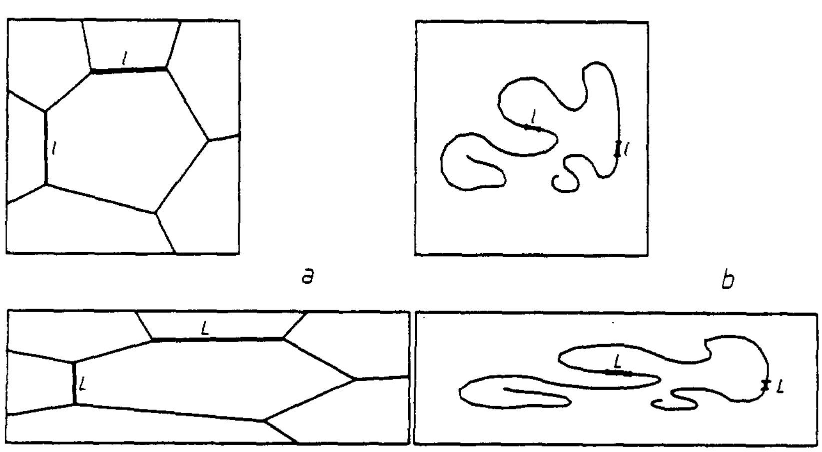

Fig. 1. Deformation of (a) polygonal array of cracks and (b) irregular curve: note change in length and orientation of line elements.

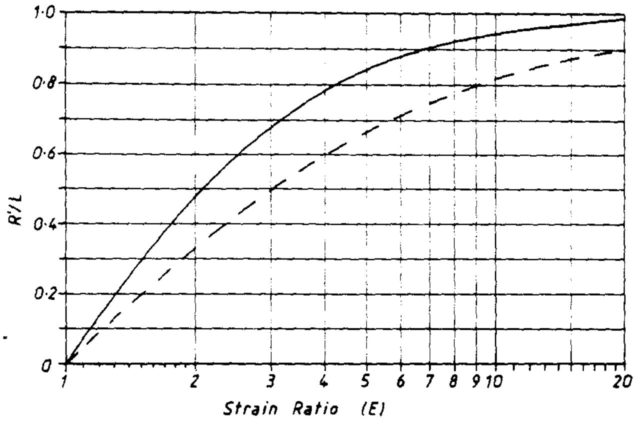

Fig. 2. Relationship between $R'/\Sigma L$ and strain ratio (E) based on analytical deformation of 1000 uniformly distributed unit vectors; corresponding relationship between R/n and E (Sanderson 1977) is shown as dashed line.

parts, close to X, will be over-represented. Thus a greater preferred orientation of line elements (R/n) will result and hence equation (1) will yield an overestimate of E. In other words simply dividing the deformed curve into equal line elements is not valid in most of the existing types of analyses.

LENGTH-WEIGHTED METHOD

To overcome these limitations it is necessary to examine the relationship between the strain ratio (E) and a length-weighted estimate of dispersion ($R'/\Sigma L$). This was obtained by considering line elements of length L and oriented at θ to an arbitrary reference direction. Each element is modelled as a vector of length L and orientation 2θ. Summation of these vectors yields a resultant of magnitude R'. The length-weighted dispersion parameter is given by $R'/\Sigma L$, where ΣL is the sum of the individual line lengths. The orientation of the resultant vector gives an estimate of $2\bar{\theta}$, where $\bar{\theta}$ is the mean orientation of the lines and will be parallel to the X-axis for an initial uniform distribution.

We have not been able to derive a simple algebraic relationship between $R'/\Sigma L$ and E, but have evaluated this by analytically deforming a uniform array of 1000 unit lines in the range 0–180°. The results are given in Table 1 and Fig. 2; in the latter, the R/n values for the corresponding unweighted vectors are given for comparison.

Panozzo (1983, 1984) gave an alternative, but more complex, procedure for analysing this problem which involved calculating the projection of each line onto an axis which, when rotated through 180°, yields maximum and minimum values proportional to the principal stretches. This method requires much more computation (but see Panozzo 1987 for a graphical solution), especially to obtain reasonable resolution of the principal axes. Both methods require unbiased samples from initial uniform distributions of lines or line segments and the data should be examined for obvious signs of departure from this assumption, e.g. non-bipolar distributions

Table 1. Relationship between strain ratio (E) and normalized length of the resultant vector ($R'/\Sigma L$) based on simulated deformation of an initial uniform distribution of 1000 lines

E	$R'/\Sigma L$	E	$R'/\Sigma L$	E	$R'/\Sigma L$
1.0	0	2.5	0.600	9	0.937
1.1	0.071	3.0	0.682	10	0.947
1.2	0.136	3.5	0.741	11	0.954
1.3	0.194	4.0	0.785	12	0.960
1.4	0.247	4.5	0.818	13	0.965
1.5	0.296	5.0	0.844	14	0.969
1.6	0.339	5.5	0.865	15	0.973
1.7	0.379	6.0	0.881	20	0.988
1.8	0.416	6.5	0.895	30	0.992
1.9	0.449	7.0	0.906	40	0.995
2.0	0.480	7.5	0.916	50	0.997
		8.0	0.924	100	0.999

and symmetry axes oblique to known strain axes (cf. R_f/ϕ methods).

COMPARISON OF LENGTH-WEIGHTED AND UNIT VECTOR ESTIMATES OF STRAIN RATIOS

An initial objective of this study was to see if length-weighting would improve strain estimates obtained from deformed lines. Data from deformed desiccation cracks sampled at various localities in the Devonian of SW Ireland were analysed by both the unit vector and length-weighting methods. Data were obtained by digitizing photographs of bedding surfaces or by direct measurement of the length and orientation of cracks in the field. The results are presented in Table 2 and Fig. 3.

In general there is a good correlation between the two methods; a paired comparison of the strain estimates showed no significant difference. Some departure at higher strains, although not statistically significant, could be due to either an underestimate of E by length-weighting or an overestimate by the unit vector method. The latter effect would be expected from undersampling of shortened cracks. We believe, however, that length-weighting does not significantly improve strain estimates for desiccation cracks. Indeed we favour the unit vector method at low strains, since the initial distribution of orientation is usually more uniform than the crack length and the method involves less field measurement. The additional information used in length-weighting is probably more than counteracted by the additional variance of the initial lengths of the cracks. Different conclusions may be reached for other material and other types of strain markers.

ANALYSIS OF LINE ELEMENTS FROM IRREGULAR CURVES

The main advantage of the length-weighting method lies in its applicability to a wider range of strain markers (see Panozzo 1984). Any irregular array of curved lines, which initially shows no preferred orientation, can be considered as being made up of connected line elements,

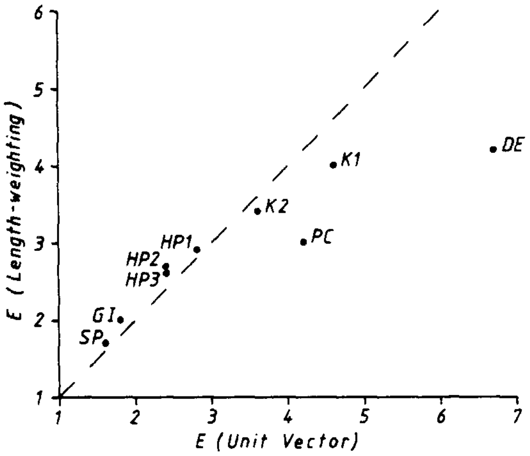

Fig. 3. Comparison of strain ratios determined by unit vector and length-weighting methods for desiccation cracks from SW Ireland (data in Table 2).

provided their length is much less than the radius of curvature of the line. When deformed these line elements change length and orientation. If they are resampled using the length-weighting technique then a valid strain estimate can be made using Table 1. All that is necessary is for the curves to be digitized with a spacing, not necessarily regular, which is much less than the radius of curvature of the deformed curve.

Obvious candidates for this form of analysis would be deformed trace fossils such as grazing trails or burrows. Figure 4(a) represents a portion of some *Chondrites* digitized from a photograph of an undeformed bedding plane in the Dingle Peninsula, W. Ireland. The data have been analytically deformed with strain ratios of 2 and 4, and the resulting ratio $R'/\Sigma L$ calculated. This produces values of $R'/\Sigma L = 0.43$ and 0.764, which correspond to strain ratios of 1.9 and 3.8, respectively; which are good estimates of the simulated strain values.

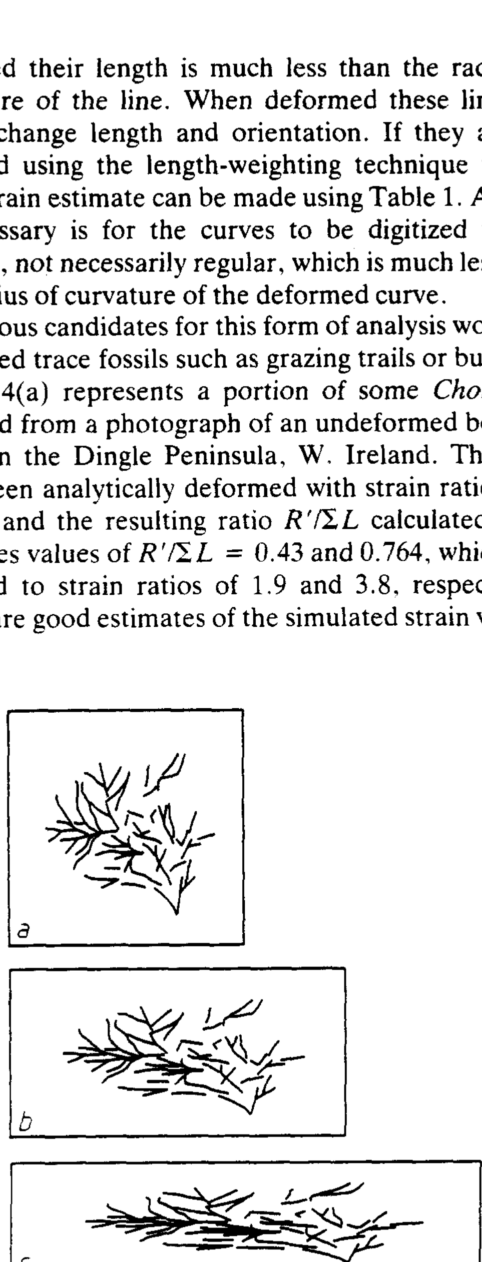

Fig. 4. Simulated deformation of a sample of *Chondrites* trace fossils from Dingle Peninsula, SW Ireland. Strain ratios of (a) 1.0, (b) 2.0 and (c) 4.0.

Table 2. Comparison of strain estimates from Devonian rocks of W Cork, Ireland, using unit vector and length weighting methods. E is the estimated strain ratio, n the sample size and R/n and $R'/\Sigma L$ are the dispersion parameters for the unit vector and length-weighting methods, respectively

			Unit vector		Length-weighted	
Locality	Grid reference	n	R/n	E	$R'/\Sigma L$	E
DE	V855323	53	0.741	6.7	0.80	4.2
K1	V832315	52	0.637	4.6	0.786	4.0
K2	V827304	32	0.572	3.6	0.731	3.4
GI	V796255	73	0.282	1.8	0.48	2.0
SP	V816269	64	0.239	1.6	0.372	1.7
PC	V564470	54	0.615	4.2	0.676	3.0
HP1	V786534	216	0.476	2.8	0.669	2.9
HP2	V786534	229	0.437	2.4	0.634	2.7
HP3	V786534	294	0.411	2.4	0.624	2.6

DISCUSSION

The dispersion of length-weighted line elements provides an additional method of analysing deformed lines which were initially randomly oriented. It also allows the random line approach to be extended to curved lines and outlines of objects under certain conditions. The method combines the two features of change in orientation and change in length of the line elements into a simple analytical procedure easily performed on a wide variety of deformed objects.

The basic assumption is that the length-weighted distribution of initial line elements is 'random'. We should consider carefully what we mean by 'random'! The essential feature is that the length-weighted orientations of the lines in the undeformed state can be considered as being sampled from a uniform distribution. More formally, if θ is the orientation of a line and L its length, then the probability density function of $L\theta$ is independent of θ, such that:

$$f(L\theta) = (2\pi)^{-1}, \quad 0 < \theta < 2\pi.$$

The initial line elements need be neither straight, randomly oriented nor of uniform length; all that is necessary is for the initial orientation of the tangents to the curve(s) to be uniformly distributed. A circle and a random array of straight or curved lines (including ellipse outlines, cf. R_f/ϕ methods) have this property. The outlines of irregular, equant clasts or grains might approximate this condition even if individuals have angular shapes. To a lesser extent, certain equant fossils (including logarithmic spirals) may meet the assumption of the method. We have tested the method by deforming various equant geometrical shapes (squares, triangles, pentagons, etc.) and if the number of independent sides exceeds 3 the results are within a few per cent of the strain ratio (note that squares and other even-sided polygons have parallel pairs of sides and thus only half of these are independent).

Since the analysis is easily performed on digitized lines (or outlines) it can be applied rapidly using a digitizing tablet linked to a computer. When applied to the closed outlines of objects there is no necessity to 'fit' arbitrary shapes (such as ellipses), nor even to calculate their 'centre of mass'. The resulting polygons could be analysed by the eigenvalue methods of Harvey & Ferguson (1981), which are themselves sensitive to the lengths of the polygon sides. The vector method presented here is simple and easy to compute, and has the additional advantage of being able to handle partial polygonal outlines. We would stress, however, that the unbiased fitting of line segments to curved outlines requires a more extensive treatment and is currently under investigation.

More complex initial ('non-random') conditions might be detected by simply displaying length-weighted histograms or rose diagrams of the line elements, even in the deformed state. This procedure is considered advisable in all cases since polymodal data can produce spurious dispersion parameters not simply related to strain.

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